Mathematics for Data Science - 1

Frequently Asked Questions

Week 1

1. **Is 0 a natural number?**
There are two conventions: some books consider 0 as a natural number, and some do not. In this course, we will consider 0 to be a natural number as is explained by the Professor in Lecture 1 on *Natural numbers and their operations.*

2. **Is 0 an even number?**
A number is *even* if it has 2 as its factor. Every number is a factor of 0. Hence, 2 is also a factor of 0. It follows that 0 is an even number.

3. **Is 1 a prime number or a composite number?**
Every prime number has exactly two factors, 1 and itself. Every composite number has more than two factors. Since 1 has only one factor, which is itself, we conclude that 1 is neither a prime number nor a composite number.

4. ***The complement of the set of prime numbers in* $\mathbb{N}$ *is the set of composite numbers.* Does this mean that 0 and 1 are composite numbers?**
Generally, when we talk about prime numbers and composite numbers, we consider natural numbers greater than or equal to 2. This is because 0 and 1 are neither prime nor composite. That is, composite numbers are the natural numbers, except 0 and 1, that are not prime.

5. **Why do we say that *irrational numbers* are dense in nature?**
We know that rational numbers are dense in nature. So, between any two rational numbers, there is always another (in fact, many) rational number. Hence, in order to prove that irrational numbers are dense, it is enough to show that between any two rational numbers, there is always an irrational number. Given two rational numbers x and y, by the property of denseness of rational numbers, there is rational number q such that $x < q < y$. Hence, we have $\frac{y-q}{2} > 0$. It implies that there

exists $n \in \mathbb{N}$ such that $\frac{y-q}{2} > \frac{1}{n} > 0$. Observe that if we choose any number which is strictly greater than 0, then we have some natural number n such that $\frac{1}{n}$ lies between that number and 0. Putting all these together we have the following inequality $x < q + \frac{\sqrt{2}}{n} < q + \frac{2}{n} < y$. Further, for any two non-zero rational numbers r_1 and r_2, $\frac{r_1}{r_2}$ is also a rational number. Suppose $\frac{\sqrt{2}}{n}$ is a rational number. Then, $\frac{\frac{\sqrt{2}}{n}}{\frac{1}{n}} = \sqrt{2}$ is also a rational number, which is not true. Hence $\frac{\sqrt{2}}{n}$ is an irrational number, which implies that, $q + \frac{\sqrt{2}}{n}$ is also an irrational number which lies between x and y. Hence, irrational numbers are dense in nature.

6. **Suppose a *divides* b, then which of the following is true: (i) a *mod* $b = 0$ (ii) b *mod* $a = 0$?**
   a divides b implies that when b is divided by a, it leaves remainder 0 which, in turn, implies that b *mod* $a = 0$.

7. **Can sets have duplicate elements?**
   Generally, if there are duplicate elements in a set, only one instance of every element is considered. The remaining instances of that same element is ignored. Duplicate elements do not affect on cardinality of a set. However, sets in which duplicate elements are considered valid are called *multisets*.

8. **What is the power set? What is the cardinality of the power set of a set having n elements?**
   Power Set: The power set of a set X is the set of all subsets of X. It is denoted as $P(X)$. For example, let $S = \{p, q, r\}$. Then, $P(S) = \{\emptyset, \{p\}, \{q\}, \{r\}, \{p, q\}, \{q, r\}, \{p, r\}, \{p, q, r\}\}$.
   In order to find the subsets of a given set S, we consider the possibility of an element belonging to a subset or not. We use 1 to denote if an element belongs to a subset and 0 to denote if an element does not belong to a subset. Given a set of 3 elements, there are 8 ways to assign 0 or 1 to each element in S, as shown in Table 1. Each combination of 0 and 1 corresponds to a unique subset of S and therefore there are 8 subsets of set S which includes empty set and the set S itself.

   Similarly, suppose we have n elements in a set D. If we assign 0 and 1 as explained, then there are 2^n many possible combinations and each

No.	assigned values to elements in S	Corresponding subset of S
1	p $\longrightarrow$ 0, q $\longrightarrow$ 0, r $\longrightarrow$ 0	$\emptyset$
2	p $\longrightarrow$ 1, q $\longrightarrow$ 0, r $\longrightarrow$ 0	{p}
3	p $\longrightarrow$ 0, q $\longrightarrow$ 1, r $\longrightarrow$ 0	{q}
4	p $\longrightarrow$ 0, q $\longrightarrow$ 0, r $\longrightarrow$ 1	{r}
5	p $\longrightarrow$ 1, q $\longrightarrow$ 1, r $\longrightarrow$ 0	{p,q}
6	p $\longrightarrow$ 0, q $\longrightarrow$ 1, r $\longrightarrow$ 1	{q,r}
7	p $\longrightarrow$ 1, q $\longrightarrow$ 0, r $\longrightarrow$ 1	{p,r}
8	p $\longrightarrow$ 1, q $\longrightarrow$ 1, r $\longrightarrow$ 1	{p,q,r}

Table 1: Computing Subsets

combination corresponds to a subset of D. It follows that there are 2^n subsets of D including empty set and the set itself. Hence, cardinality of the power set of a set having n elements is 2^n.

9. **What is the set comprehension form of the set of rational numbers in reduced form?**
   The set of rational numbers in reduced form is

$$\{\tfrac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0, g.c.d(p, q) = 1\}$$

10. **What is an anti-symmetric relation?**
    An anti-symmetric relation R is one in which for any ordered pair (x, y) in R such that $x \neq y$, the ordered pair (y, x) does not belong to R.
    For example, let $R = \{(1, 2), (2, 4), (3, 5), (4, 4)\}$. R is an anti-symmetric relation because $(1, 2) \in R$ but $(2, 1) \notin R$. Similarly $(2, 4), (3, 5) \in R$ but $(4, 2), (5, 3) \notin R$.
    We give another example below. Let $W = \{x \mid x \text{ is a person }\}$. Let $x, y \in W$. Define a relation T on W as follows: $(x, y) \in T$ if and only if x is taller than y. If x is taller than y, then y cannot be taller than x. It follows that $(y, x) \notin T$. Hence, T is an anti-symmetric relation.

11. **What is the syllabus for the qualifier exam?**
    The syllabus for the qualifier exam is contents of Week 1 to Week 4. The Week 1 videos are already available on onlinedegree.iitm.ac.in. The remaining contents for Week 1 will be released on Oct 5th. The contents for Week 2 will be released on Oct 12th, Week 3 on Oct 19th and Week

4 on Oct 26th. (Any change in date of release will be informed in advance.)

12. **Can we get slides/pdf for the lectures?**
    Yes, the transcript notes of lectures along with the slides used by the instructors will be provided during weekly content release.

13. **Are there any reference books for Mathematics for Data Science-1 course?**
    For Mathematics for Data Science - I course, we do not have any prescribed books. The contents are predominantly from the NCERT classes X,XI,XII textbooks. The contents that are released from the course portal will be the primary material for this course. Apart from this, the textbooks or any supplementary problem solving books based on NCERT X, XI, XII classes may be used for improving problem solving skills and speed.